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# Form factors, branching ratio and forward–backward asymmetry in $B \to K_1 \ell^+ \ell^-$ decays

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**Abstract.** We study long-distance effects in the rare exclusive semileptonic decays $B \to K_1 \ell^+ \ell^-$, where $K_1$ is the axial vector meson. The form factors describing the meson transition amplitudes of the effective Hamiltonian are calculated using the Ward identities, which are then used to calculate the branching ratio and the forward–backward asymmetry in these decay modes. The zero of the forward–backward asymmetry is of special interest and provides us with a precision test of the standard model.

## 1 Introduction

The investigation of rare semileptonic decays of the $B$ meson induced by the flavor-changing neutral-current (FCNC) transitions $b \to s$ provides potentially stringent tests of the standard model (SM) in flavor physics. In the SM these FCNC transitions are not allowed at tree level but are induced by the Glashow–Iliopoulos–Miani (GIM) amplitudes [1] at loop level. Additionally these are also suppressed in the SM due to their dependence on the weak mixing angles of the quark-flavor rotation matrix – the Cabibbo–Kobayashi–Maskawa (CKM) matrix [2, 3]. These two circumstances make the FCNC decays relatively rare and hence important for the presence of new physics, i.e., physics beyond SM.

The experimental observation of the inclusive [4] and exclusive [5, 6] decays $B \to X_s \gamma$ and $B \to K^* \gamma$ has prompted a lot of theoretical interest in rare $B$ meson decays. However, in case of exclusive decays any reliable extraction of the perturbative (short-distance) effects encoded in the Wilson coefficients of the effective Hamiltonian [7–15] requires an accurate separation of the non-perturbative (long-distance contributions), which therefore should be known with high accuracy. The theoretical investigation of these contributions encounters the problem of describing the hadron structure, which provides the main uncertainty in the predictions of exclusive rare decays. In exclusive $B \to K, K^*$ decays the long-distance effects in the meson transition amplitude of the effective Hamiltonian are encoded in the meson transition form factors. Many exclusive $B \to K\,(K^*)\,\ell^+\ell^-$ [16–34], $B \to \gamma \ell^+ \ell^-$ [35–40], and $B \to \ell^+\ell^-$ [41–47] processes based on $b \to s\,(d)\,\ell^+\ell^-$ have been studied in the literature and many frameworks have been applied to the description of meson transition form factors: among them those worth mentioning are constituent quark models, QCD sum rules, lattice QCD, approaches based on heavy quark symmetry and analytical constraints. Many observables, like the forward–backward (FB) asymmetry, single and double lepton polarization asymmetries associated with the final state leptons, have been extensively studied for quite some time for quark level processes $b \to s\,(d)\,\ell^+\ell^-$.

Recently, Belle [48] has announced the first measurement of $B \to K_1^+(1270)\gamma$

$$\mathcal{B}(B^+ \to K_1^+ \gamma) = (4.28 \pm 0.94 \pm 0.43) \times 10^{-5}\,, \qquad (1)$$

after which these radiative decays became a topic of prime interest and there is a lot of theoretical progress [49–52]. In this paper we study the semileptonic $B$ meson decay $B \to K_1 \ell^+ \ell^-$ using the framework of Gilani et al. [53], with $K_1$ an axial vector meson. The axial vector mesons is distinguished from vector mesons by an extra $\gamma_5$ in the gamma structure of the decay amplitude (DA) and some non-perturbative parameters. But the presence of the extra $\gamma_5$ does not alter the calculation except for the switching of vector to axial vector form factors and vice versa. As mentioned earlier, the theoretical understanding of exclusive decays is complicated mainly due to non-perturbative form factors entering in the long-distance non-perturbative contributions. The aim of this work is to relate the various form factors in a model independent way through the Ward identities. This enables us to make a clear separation between non-pole and pole type contributions; the $q^2 \to 0$ behavior of the former is known in terms of a universal function $\xi_\perp(0) \equiv g_+(0)$ introduced in the large energy effective theory (LEET) of heavy ($B$) to light ($K_1$) form factors [51, 52]. The residue of the pole is then determined in a self-consistent way in terms of $g_+(0)$ or $\xi_\perp(0)$, which will give information on the couplings of $B^*(1^-)$ and $B_A^*(1^+)$

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with the $BK_1$ channel. The form factors are then determined in terms of known parameters like $g_+(0)$ and the masses of the particles involved, which are then used to calculate the branching ratio and forward–backward asymmetry for these decays.

This paper is organized as follows: in Sect. 2 we introduce the effective Hamiltonian formalism of the semileptonic $B$ meson decays and we will write down the matrix elements for $B \to K_1 \ell^+ \ell^-$ decays. Section 3 discusses the Ward identities and develops the relationship between the form factors, which results in a reduction of a number of unknown quantities. The form factors thus obtained are used for the calculation of the decay width and the forward–backward asymmetry. Finally, in the last section we summarize our conclusions.

## 2 Effective Hamiltonian

At quark level the decay $B \to K_1 \ell^+ \ell^-$ is similar to the one studied in, for example, [16]. The basic transition $b \to s\ell^+\ell^-$ is described by the effective Hamiltonian given here:

$$H_{\text{eff}} = -4\frac{G_{\text{F}}}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu) , \tag{2}$$

where the $O_i$ are four local quark operators and the $\dot{C}_i$ are Wilson coefficients calculated in the naive dimensional regularization (NDR) scheme [54].

One can write the above Hamiltonian in the following free quark decay amplitude:

$$\begin{aligned}\mathcal{M}(b \to s\ell^+\ell^-) = \frac{G_{\text{F}}\alpha}{\sqrt{2}\pi} V_{tb} V_{ts}^* \Big\{ & C_9^{\text{eff}} \left[\bar{s}\gamma_\mu L b\right] \left[\bar{\ell}\gamma^\mu \ell\right] \\ & + C_{10} \left[\bar{s}\gamma_\mu L b\right] \left[\bar{\ell}\gamma^\mu \gamma^5 \ell\right] \\ & - 2\hat{m}_b C_7^{\text{eff}} \left[\bar{s}\mathrm{i}\sigma_{\mu\nu}\frac{\hat{q}^\nu}{\hat{s}} R b\right] \left[\bar{\ell}\gamma^\mu \ell\right] \Big\} ,\end{aligned} \tag{3}$$

with $L/R \equiv \frac{(1\mp\gamma_5)}{2}$, and $s = q^2$, which is just the momentum transfer from heavy to light meson. The amplitude given in (3) is a free quark decay amplitude, which contains a certain long-distance effect from the matrix element of local quark operators, $\langle l^+ l^- s\,| O_i |\, b\rangle$, $1 \le i \le 6$, which is usually reabsorbed into the redefinition of the short-distance Wilson coefficients. Specifically, for the exclusive decays, the effective coefficients of the operator $O_9 = \frac{e^2}{16\pi^2}(\bar{s}\gamma_\mu L b)(\bar{l}\gamma^\mu l)$ can be written as

$$C_9^{\text{eff}} = C_9 + Y(\hat{s}) , \tag{4}$$

where the perturbatively calculated result of $Y(\hat{s})$ is [54, 55]

$$\begin{aligned}Y_{\text{pert}}(\hat{s}) = {} & g(\hat{m}_c, \hat{s})(3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) \\ & - \frac{1}{2} g(1, \hat{s})(4C_3 + 4C_4 + 3C_5 + C_6) \\ & - \frac{1}{2} g(0, \hat{s})(C_3 + 3C_4) \\ & + \frac{2}{9}(3C_3 + C_4 + 3C_5 + C_6) .\end{aligned} \tag{5}$$

For the values of the Wilson coefficients and the explicit expressions of the $g$ appearing in (5) we refer to [54, 55]. The hat denotes normalization in terms of the $B$ meson mass [16].

## 3 Matrix elements and Ward identities

The exclusive decays $B \to K_1 \ell^+ \ell$ involve the hadronic matrix elements of the quark operators in (3) between $B$ and $K_1$. These can be parameterized in terms of form factors, which are scalar functions of the four momentum square $(q^2 = (p_B - p_{K_1})^2)$. For the process we are considering, there are seven form factors like the transition of the pseudoscalar to the vector meson. The non-vanishing matrix elements are

$$\begin{aligned}\langle K_1(k,\varepsilon) | V_\mu | B(p)\rangle = {} & \mathrm{i}\varepsilon_\mu^* (M_B + M_{K_1}) V_1(s) \\ & - (p+k)_\mu (\varepsilon^* \cdot q) \frac{V_2(s)}{M_B + M_{K_1}} \\ & - q_\mu (\varepsilon \cdot q) \frac{2M_{K_1}}{s} \left[V_3(s) - V_0(s)\right] ,\end{aligned} \tag{6}$$

$$\langle K_1(k,\varepsilon) | A_\mu | B(p)\rangle = \frac{2\mathrm{i}\epsilon_{\mu\nu\alpha\beta}}{M_B + M_{K_1}} \varepsilon^{*\nu} p^\alpha k^\beta A(s) , \tag{7}$$

where $V_\mu = \bar{s}\gamma_\mu b$ and $A_\mu = \bar{s}\gamma_\mu\gamma_5 b$ are the vector and axial vector currents respectively and $\varepsilon_\mu^*$ is the polarization vector for the final state axial vector meson. In (6) we have

$$V_3(s) = \frac{M_B + M_{K_1}}{2M_{K_1}} V_1(s) - \frac{M_B - M_{K_1}}{2M_{K_1}} V_2(s) , \tag{8}$$

with

$$V_3(0) = V_0(0) .$$

In addition to the above form factors there are also some penguin form factors; these are

$$\begin{aligned}& \langle K_1(k,\varepsilon) | \bar{s}\mathrm{i}\sigma_{\mu\nu} q^\nu b | B(p)\rangle \\ & = \left[\left(M_B^2 - M_{K_1}^2\right)\varepsilon_\mu - (\varepsilon \cdot q)(p+k)_\mu\right] F_2(s) \\ & \quad + (\varepsilon^* \cdot q)\left[q_\mu - \frac{s}{M_B^2 - M_{K_1}^2}(p+k)_\mu\right] F_3(s) ,\end{aligned} \tag{9}$$

$$\langle K_1(k,\varepsilon) | \bar{s}\mathrm{i}\sigma_{\mu\nu} q^\nu \gamma_5 b | B(p)\rangle = -\mathrm{i}\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu} p^\alpha k^\beta F_1(s) , \tag{10}$$

with

$$F_1(0) = 2F_2(0) .$$

The various form factors appearing in (6)–(10) can be related by the Ward identities as follows [53, 58, 59]:

$$\begin{aligned}& \langle K_1(k,\varepsilon) | \bar{s}\mathrm{i}\sigma_{\mu\nu} q^\nu b | B(p)\rangle \\ & = -(m_b + m_s) \langle K_1(k,\varepsilon) | \bar{s}\gamma_\mu b | B(p)\rangle ,\end{aligned} \tag{11}$$

$$\begin{aligned}& \langle K_1(k,\varepsilon) | \bar{s}\mathrm{i}\sigma_{\mu\nu} q^\nu \gamma_5 b | B(p)\rangle \\ & = (m_b - m_s) \langle K_1(k,\varepsilon) | \bar{s}\gamma_\mu\gamma_5 b | B(p)\rangle \\ & \quad + (p+k)_\mu \langle K_1(k,\varepsilon) | \bar{s}\gamma_5 b | B(p)\rangle .\end{aligned} \tag{12}$$

Now we make the heavy quark approximation and compare coefficients of $\varepsilon^*_\mu$ and $q_\mu$ on both sides. In the heavy quark approximation we need not to compare the coefficients $(p+k)_\mu$. Using (6)–(10) in (11) and (12), we get the following relationship between the form factors:

$$F_1(s) = -\frac{(m_b - m_s)}{M_B + M_{K_1}} 2A(s)\,, \tag{13}$$

$$F_2(s) = -\frac{(m_b + m_s)}{M_B - M_{K_1}} V_1(s)\,, \tag{14}$$

$$F_3(s) = \frac{2M_{K_1}}{s} (m_b + m_s) \left[V_3(s) - V_0(s)\right]. \tag{15}$$

These are model independent results derived by using Ward identities. The universal normalization of the above form factors at $q^2 = s = 0$ is obtained by defining [53]

$$\begin{aligned} &\left\langle K_1(k,\varepsilon) \left| \bar{s} \mathrm{i} \sigma^{\alpha\beta} \gamma^5 b \right| B(p) \right\rangle \\ &= -\mathrm{i}\epsilon^{\alpha\beta\rho\sigma} \varepsilon^*_\rho \left[(p+k)_\rho g_+ + q_\sigma g_-\right] - (q\cdot\varepsilon^*)\epsilon^{\alpha\beta\rho\sigma}(p+k)_\rho q_\sigma h \\ &\quad - \mathrm{i}\left[q^\alpha \epsilon^{\beta\rho\sigma\tau} \varepsilon^*_\rho (p+k)_\sigma q_\tau - \alpha \leftrightarrow \beta\right] h_1. \end{aligned} \tag{16}$$

Using the Dirac identity

$$\sigma_{\mu\nu}\gamma^5 = -\frac{\mathrm{i}}{2}\epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta} \tag{17}$$

in (16), one can write

$$\begin{aligned} &\langle K_1(k,\varepsilon) \left| \bar{s} \mathrm{i} \sigma_{\mu\nu} q^\nu b \right| B(p)\rangle \\ &= \varepsilon^*_\mu \left[\left(M_B^2 - M_{K_1}^2\right) g_+ + s g_-\right] - (q\cdot\varepsilon^*)\left[(p+k)_\mu g_+ + q_\mu g_-\right] \\ &\quad + (q\cdot\varepsilon^*)\left[s(p+k)_\mu - (M_B^2 - M_{K_1}^2) q_\mu\right] h\,. \end{aligned} \tag{18}$$

Comparing the coefficients of $q_\mu$, $\varepsilon^*_\mu$ and $\epsilon_{\mu\nu\alpha\beta}$ from (9), (10), (16) and (18), we get

$$F_1(s) = 2\left[g_+(s) - s h_1\right], \tag{19}$$

$$F_2(s) = g_+ + \frac{s}{M_B^2 - M_{K_1}^2} g_-\,, \tag{20}$$

$$F_3(s) = -g_- - \left(M_B^2 - M_{K_1}^2\right) h\,. \tag{21}$$

The above results ensure that $F_1(0) = 2F_2(0)$. In terms of $g_+$, $g_-$ and $h$, the form factors become

$$\begin{aligned} A(s) &= \frac{M_B + M_{K_1}}{m_b - m_s}\left[g_+(s) - s h_1\right], \\ V_1(s) &= -\frac{M_B - M_{K_1}}{m_b + m_s}\left[g_+ + \frac{s}{M_B^2 - M_{K_1}^2} g_-\right], \\ V_2(s) &= -\left(\frac{M_B + M_{K_1}}{m_b + m_s}\right)\left[g_+(s) - s h\right] - \frac{2M_{K_1}}{M_B - M_{K_1}} V_0(s)\,. \end{aligned} \tag{22}$$

By looking at the above expressions one can see that the normalization of the above form factors $A$ and $V_1$ at $s = 0$ is determined by the single constant $g_+(0)$, whereas that of $V_2$ is determined by $g_+(0)$ and $V_0(s)$.

### 3.1 Pole contributions

The pole contribution for $B$ to $\rho$ has been studied in detail by Gilani et al. [53]. This remains the same for the $B$ to $K_1$ transition and again only $h_1$, $g_-$, $h$ and $V_0$ get pole contributions from $B^*(1^-)$, $B^*_A(1^+)$ and $B(0^-)$ mesons, whereas $g_+$, $g_-$ and $V_0(s)$ get their contribution from the quark triangle graph. These are given by

$$\begin{aligned} h_1|_{\text{pole}} &= -\frac{1}{2}\frac{g_{B^*BK_1}}{M_{B^*}^2}\frac{f_T^{B^*}}{1 - s/M_{B^*}^2} = \frac{R_V}{M_{B^*}^2}\frac{1}{1 - s/M_{B^*}^2}\,, \\ g_-|_{\text{pole}} &= -\frac{g_{B_A^*BK_1}}{M_{B_A^*}^2}\frac{f^{B_A^*}}{1 - s/M_{B_A^*}^2} = \frac{R_A^S}{M_{B_A^*}^2}\frac{1}{1 - s/M_{B_A^*}^2}\,, \\ h|_{\text{pole}} &= \frac{1}{2}\frac{f_{B_A^*BK_1}}{M_{B_A^*}^2}\frac{f_T^{B_A^*}}{1 - s/M_{B_A^*}^2} = \frac{R_A^D}{M_{B_A^*}^2}\frac{1}{1 - s/M_{B_A^*}^2}\,, \\ V_0(s)|_{\text{pole}} &= \frac{g_{BBK_1}}{M_{K_1}} f_B \frac{s/M_B^2}{1 - s/M_B^2} = R_0 \frac{s/M_B^2}{1 - s/M_B^2}\,, \end{aligned} \tag{23}$$

where $R_V$, $R_A^S$, $R_A^D$ and $R_0$ are related to the coupling constants $g_{B^*BK_1}$, $g_{B_A^*BK_1}$, $f_{B_A^*BK_1}$ and $g_{BBK_1}$, respectively. One can find details in [53]. Thus one can write

$$A(s) = \left(\frac{M_B + M_{K_1}}{m_b - m_s}\right)\left(g_+(s) - R_V \frac{s}{M_{B^*}^2}\left(\frac{1}{1 - s/M_{B^*}^2}\right)\right), \tag{24}$$

$$\begin{aligned} V_1(s) = &-\left(\frac{M_B - M_{K_1}}{m_b + m_s}\right)\left(g_+(s) + \frac{s}{M_B^2 - M_{K_1}^2}\tilde{g}_-\right. \\ &\left. + \frac{R_A^S}{M_B^2 - M_{K_1}^2}\frac{s}{M_{B_A^*}^2}\left(\frac{1}{1 - s/M_{B_A^*}^2}\right)\right), \end{aligned} \tag{25}$$

$$\begin{aligned} V_2(s) = &-\left(\frac{M_B + M_{K_1}}{m_b + m_s}\right)\left[g_+(s) - \frac{s}{M_{B_A^*}^2} R_A^D \frac{1}{1 - s/M_{B_A^*}^2}\right] \\ &- \frac{2M_{K_1}}{M_B - M_{K_1}} V_0(s)\,. \end{aligned} \tag{26}$$

The behavior of $g_+(s)$, $\tilde{g}_-(s)$ and $V_0(s)$ near $s \to 0$ is known from LEET and their form is [53]

$$g_+(s) = \frac{\xi_\perp(0)}{\left(1 - s/M_B^2\right)^2} = -\tilde{g}_-(s)\,, \tag{27}$$

$$V_0(s) = \left(1 - \frac{M_{K_1}^2}{M_B E_{K_1}}\right)\xi_\parallel(s) + \frac{M_{K_1}}{M_B}\xi_\perp(s)\,. \tag{28}$$

At $s \to 0$

$$V_0(0) = \frac{M_B^2 - M_{K_1}^2}{M_B^2 + M_{K_1}^2}\xi_\parallel(0) + \frac{M_{K_1}}{M_B}\xi_\perp(0)\,, \tag{29}$$

$$E_{K_1} = \frac{M_B}{2}\left(1 - \frac{s}{M_B^2} + \frac{M_{K_1}^2}{M_B^2}\right), \tag{30}$$

$$g_+(0) = \xi_\perp(0)\,. \tag{31}$$

The pole terms in the relations (24)–(26) are expected to dominate near $s = M_{B^*}^2$ or $M_{B_A^*}^2$. On the other hand the relations obtained from the Ward identities are expected to hold for $s$ much below the resonance region. The above behavior, near $s = 0$, and that near the pole [53] suggests that

$$F(s) = \frac{F(0)}{(1 - s/M^2)(1 - s/M'^2)}, \tag{32}$$

where $M^2$ is $M^2_{B^*}$ or $M^2_{B^*_A}$, and $M'$ is the radial excitation of $M$. This parameterization not only takes into account the corrections to single pole dominance, as suggested by the dispersion relation approach [58–60] but also corrections of off-mass-shell-ness of the couplings of $B^*$ or $B^*_A$ with the $BK_1$ channel.

Since $g_+(s)$ and $\tilde{g}_-(s)$ have no pole at $s = M^2_{B^*}$, we get

$$A(s)\left(1 - \frac{s}{M^2_{B^*}}\right)\Bigg|_{s=M^2_{B^*}} = R_V \left(\frac{M_B + M_{K_1}}{m_b - m_s}\right).$$

This gives, using the parametrization (32),

$$\begin{aligned} R_V \equiv -\frac{1}{2} g_{B^* B K_1} f_T^{B^*} &= -\frac{1}{2} g_{B^* B K_1} f_B \\ &= -\frac{g_+(0)}{1 - M^2_{B^*}/M'^2_{B^*}}. \end{aligned} \tag{33}$$

Similarly,

$$R_A^D \equiv \frac{1}{2} f_{B^*_A B K_1} f_T^{B^*_A} = -\frac{g_+(0)}{1 - M^2_{B^*_A}/M'^2_{B^*_A}}. \tag{34}$$

For a detailed derivation and discussion of these relations we will refer to [53]. We cannot use the parametrization given in (32) for $V_1(s)$, since near $s = 0$, $V_1(s)$ behaves as $g_+(s)\left[1 - s/(M_B^2 - M_{K_1}^2)\right]$ [c.f. (25) and (27)]. This suggests the following:

$$V_1(s) = \frac{g_+(0)}{\left(1 - s/M^2_{B^*_A}\right)\left(1 - s/M'^2_{B^*_A}\right)} \left(1 - \frac{s}{M_B^2 - M_{K_1}^2}\right). \tag{35}$$

Until now we have expressed everything in terms of $g_+(0)$, which is the only unknown in the calculation. After the first announcement of Belle [48] for the decay $B \to K_1 \gamma$, the value of $g_+(0)$ has been extracted to be [49–52]

$$g_+(0) = \xi_\perp(0) = 0.32 \pm 0.03. \tag{36}$$

Using $f_B = 180\,\mathrm{MeV}$ we have the prediction from (33) that

$$g_{B^* B K_1} = 15.42\,\mathrm{GeV}^{-1}. \tag{37}$$

Similarly, the $S$ and $D$ wave couplings are predicted to be

$$g_{B^*_A B K_1} = 3.17 f_{B^*_A B K_1}\,\mathrm{GeV}^2. \tag{38}$$

The different values of the $F(0)$ are

$$A(0) = \left(\frac{M_B + M_{K_1}}{m_b - m_s}\right) g_+(0), \tag{39}$$

$$V_1(0) = -\left(\frac{M_B - M_{K_1}}{m_b + m_s}\right) g_+(0), \tag{40}$$

$$V_2(0) = -\left(\frac{M_B + M_{K_1}}{m_b + m_s}\right) g_+(0) - \frac{2M_{K_1}}{M_B - M_{K_1}} V_0(0), \tag{41}$$

where $g_+(0)$ is the same as in (36). The calculation of the numerical values of $A(0)$ and $V_1(0)$ is very trivial, but to go for $V_2(0)$, we have to know the value of $V_0\,(0)$. Although LEET does not give any relationship between $\xi_\parallel(0)$ and $\xi_\perp(0)$, due to some numerical coincidence in the LCSR expressions for $\xi_\parallel(0)$ and $\xi_\perp(0)$ [61],

$$\xi_\parallel(0) \simeq \xi_\perp(0) = g_+(0), \tag{42}$$

from (29) we have

$$V_0(0) = 1.13 g_+(0). \tag{43}$$

Thus the final expressions of the form factors that we shall use for the numerical work are

$$\begin{aligned} A\,(s) &= \frac{A\,(0)}{(1 - s/M_B^2)(1 - s/M'^2_B)}, \\ V_1(s) &= -\frac{V_1(0)}{\left(1 - s/M^2_{B^*_A}\right)\left(1 - s/M'^2_{B^*_A}\right)} \left(1 - \frac{s}{M_B^2 - M_{K_1}^2}\right), \\ V_2(s) &= -\frac{\tilde{V}_2(0)}{\left(1 - s/M^2_{B^*_A}\right)\left(1 - s/M'^2_{B^*_A}\right)} \\ &\quad - \frac{2M_{K_1}}{M_B - M_{K_1}} \frac{V_0(0)}{(1 - s/M_B^2)(1 - s/M'^2_B)}, \end{aligned} \tag{44}$$

where

$$\begin{aligned} A(0) &= (0.52 \pm 0.05), \\ V_1(0) &= -(0.24 \pm 0.02), \\ \tilde{V}_2(0) &= -(0.39 \pm 0.03). \end{aligned} \tag{45}$$

## 4 Decay distribution and forward–backward asymmetry

In this section we define the decay rate distribution that we shall use for the phenomenological analysis . Following the notation from [16] we can write from (3)

$$\mathcal{M} = \frac{G_F \alpha}{2\sqrt{2}\pi} V_{tb} V_{ts}^* m_B \left[\mathcal{T}_\mu^1 \left(\bar{l}\gamma^\mu l\right) + \mathcal{T}_\mu^2 \left(\bar{l}\gamma^\mu \gamma^5 l\right)\right], \tag{46}$$

where

$$\begin{aligned}\mathcal{T}_\mu^1 &= A(\hat{s})\,\varepsilon_{\mu\rho\alpha\beta}\epsilon^{*\rho}\hat{p}_B^\alpha \hat{p}_{K_1}^\beta - \mathrm{i}B(\hat{s})\,\epsilon_\mu^* \\ &\quad + \mathrm{i}C(\hat{s})\,(\epsilon^*\cdot\hat{p}_B)\,\hat{p}_{h\mu} + \mathrm{i}D(\hat{s})\,(\epsilon^*\cdot\hat{p}_B)\,\hat{q}_\mu\,, \quad (47)\\ \mathcal{T}_\mu^2 &= E(\hat{s})\,\varepsilon_{\mu\rho\alpha\beta}\epsilon^{*\rho}\hat{p}_B^\alpha \hat{p}_{K_1}^\beta - \mathrm{i}F(\hat{s})\,\epsilon_\mu^* \\ &\quad + \mathrm{i}G(\hat{s})\,(\epsilon^*\cdot\hat{p}_B)\,\hat{p}_{h\mu} + \mathrm{i}H(\hat{s})\,(\epsilon^*\cdot\hat{p}_B)\,\hat{q}_\mu\,. \quad (48)\end{aligned}$$

The definition of the different momenta involved are defined in [16], where the auxiliary functions are

$$\begin{aligned}A(\hat{s}) &= -\frac{2A(\hat{s})}{1+\hat{M}_{K_1}}C_9^{\mathrm{eff}}(\hat{s}) + \frac{2\hat{m}_b}{\hat{s}}C_7^{\mathrm{eff}}F_1(\hat{s})\,,\\ B(\hat{s}) &= \left(1+\hat{M}_{K_1}\right)\left[C_9^{\mathrm{eff}}(\hat{s})V_1(\hat{s}) + \frac{2\hat{m}_b}{\hat{s}}C_7^{\mathrm{eff}}\left(1-\hat{M}_{K_1}\right)\right],\\ C(\hat{s}) &= \frac{1}{\left(1-\hat{M}_{K_1}^2\right)}\left\{C_9^{\mathrm{eff}}(\hat{s})V_2(\hat{s})\right.\\ &\quad \left. +2\hat{m}_b C_7^{\mathrm{eff}}\left[F_3(\hat{s}) + \frac{1-\hat{M}_{K_1}^2}{\hat{s}}F_2(\hat{s})\right]\right\},\\ D(\hat{s}) &= \frac{1}{\hat{s}}\left[\left(C_9^{\mathrm{eff}}(\hat{s})(1+\hat{M}_{K_1})V_1(\hat{s}) - (1-\hat{M}_{K_1})V_2(\hat{s})\right.\right.\\ &\quad \left.\left. -2\hat{M}_{K_1}V_0(\hat{s})\right) - 2\hat{m}_b C_7^{\mathrm{eff}}F_3(\hat{s})\right],\\ E(\hat{s}) &= -\frac{2A(\hat{s})}{1+\hat{M}_{K_1}}C_{10}\,,\\ F(\hat{s}) &= \left(1+\hat{M}_{K_1}\right)C_{10}V_1(\hat{s})\,,\\ G(\hat{s}) &= \frac{1}{1+\hat{M}_{K_1}}C_{10}V_2(\hat{s})\,,\\ H(\hat{s}) &= \frac{1}{\hat{s}}\left[C_{10}(\hat{s})(1+\hat{M}_{K_1})V_1(\hat{s}) - (1-\hat{M}_{K_1})V_2(\hat{s})\right.\\ &\quad \left. -2\hat{M}_{K_1}V_0(\hat{s})\right]. \quad (49)\end{aligned}$$

The differential decay rate for $B \to K^*\mu^+\mu^-$ can be expressed in terms of these auxiliary functions in [16] and this remains the same for $B \to K_1\mu^+\mu^-$ with the obvious replacements. Integration on $\hat{s}$ in the range

$$(2\hat{m}_l)^2 \le \hat{s} \le (1-\hat{m}_{K_1})^2\,, \quad (50)$$

with $\hat{m}_l = m_l/m_B$, and using $\tau_{B^0} = (1.530 \pm 0.009) \times 10^{-12}$ s, the branching ratio is

$$\mathcal{B}\left(B \to K_1\mu^+\mu^-\right) = 0.9^{+0.11}_{-0.14} \times 10^{-7}\,.$$

The above value of the branching ratio is for the case that we do not include $Y(\hat{s})$ in (4). The error in the value reflects the uncertainty from the form factors, and also is due to the variation of the input parameters like the CKM matrix elements, decay constant of $B$ meson and masses as defined in Table 1.

Now if we include the value of $Y(\hat{s})$ the central value of the branching ratio reduces to

$$\mathcal{B}\left(B \to K_1\mu^+\mu^-\right) = 0.72 \times 10^{-7}\,.$$

On including $Y(\hat{s})$ the behavior of the differential decay rate as a function of $\hat{s}$ is shown in Fig. 1. The solid line denotes the theoretical prediction with the input parameters taken at their central values, while the band between the two dashed lines shows the uncertainty from the input parameters. In our numerical analysis we have considered only the final state leptons as being muons. Our reason for choosing this is due to the extreme difficulty in detecting electrons in the final state, and also the branching ratio $B \to K_1\ell^+\ell^-$ becomes small with the SM for the $\tau$ in the final state.

**Table 1.** Default value of the input parameters used in the calculation

| | |
|---|---|
| $m_W$ | 80.41 GeV |
| $m_Z$ | 91.1867 GeV |
| $\sin^2\theta_{\mathrm{W}}$ | 0.2233 |
| $m_c$ | 1.4 GeV |
| $m_{b,\mathrm{pole}}$ | $4.8 \pm 0.2$ GeV |
| $m_t$ | $173.8 \pm 5.0$ GeV |
| $\alpha_{\mathrm{s}}(m_Z)$ | $0.119 \pm 0.0058$ |
| $f_B$ | $(200 \pm 30)$ MeV |
| $\lvert V_{ts}^* V_{tb}\rvert$ | 0.0385 |

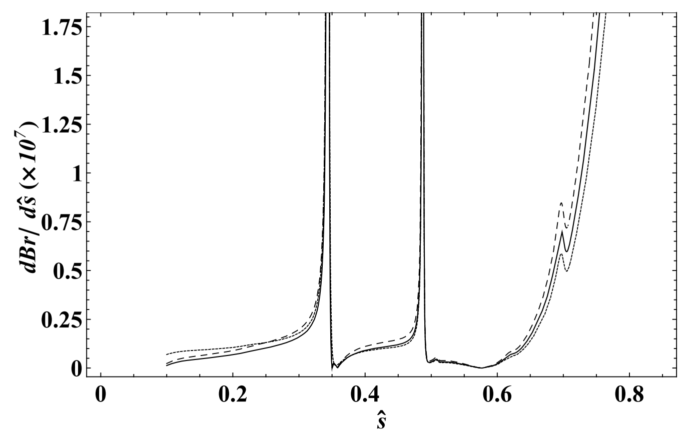


**Fig. 1.** The differential decay rate as a function of $\hat{s}$ is plotted using the form factors calculated by using the Ward identities. The resonanant $c\bar{c}$ states are parameterized as in [54, 55]. Here the *solid line* denotes the theoretical predictions with the input parameters taken at their central values, while the *dashed* (*dotted*) *line* is for max. (min) value of the input parameters

The differential forward–backward asymmetry for $B \to K_1\mu^+\mu^-$ reads as follows [16]:

$$\begin{aligned}\frac{\mathrm{d}\mathcal{A}_{\mathrm{FB}}}{\mathrm{d}\hat{s}} &= \frac{G_{\mathrm{F}}^2\alpha^2 m_B^5}{2^{10}\pi^5}\,\lvert V_{ts}^* V_{tb}\rvert^2\\ &\quad \times \hat{s}\hat{u}(\hat{s})\left[\mathrm{Re}\left(BE^*\right) + \mathrm{Re}\left(AF^*\right)\right]\,, \quad (51)\end{aligned}$$

where

$$\begin{aligned}\hat{u}(\hat{s}) &= \sqrt{\lambda\left(1-4\frac{\hat{m}_l^2}{\hat{s}}\right)}\,,\\ \lambda &\equiv \lambda\left(1, \hat{m}_{K_1}^2, \hat{s}\right)\\ &= 1 + \hat{m}_{K_1}^4 + \hat{s}^2 - 2\hat{s} - 2\hat{m}_{K_1}^2(1+\hat{s})\,. \quad (52)\end{aligned}$$

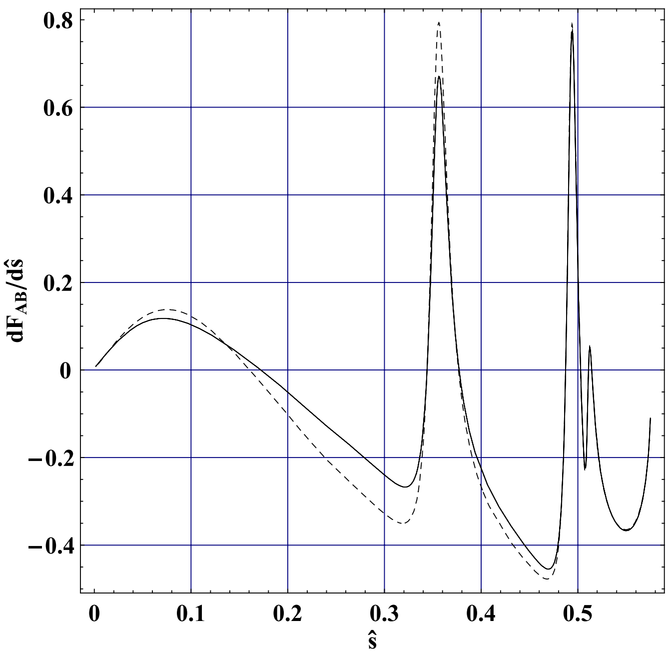


**Fig. 2.** The forward–backward (FB) asymmetry as a function of $\hat{s}$ is plotted using the form factors calculated by using the Ward identities. The resonant $c\bar{c}$ states are parameterized as in [54, 55]. The *dashed* (*solid*) *line* is for the central (max.) value of the input paramters

The variable $\hat{u}$ corresponds to $\theta$, the angle between the momenta of the $B$ meson and the positively charged lepton in the dilepton c.m. system frame. The behavior of the forward–backward asymmetry in $B \to K_1 \mu^+ \mu^-$ decay as a function of $\hat{s}$ is shown in Fig. 2. Contrary to the branching ratio, the forward–backward asymmetry is less sensitive to the input parameters as is clear from Fig. 2. For the zero-point of the forward–backward asymmetry in the standard model, we get $\hat{s} = (0.16 + 0.01)$ and $\left(s = (4.46 + 0.27)\ \text{GeV}^{-2}\right)$.

## 5 Conclusions

We have studied $B \to K_1 \ell^+ \ell^-$ decay using the Ward identities. The form factors have been calculated and we found that their normalization is essentially determined by the single constant $g_+(0)$, which has the value $g_+(0) = 0.32 \pm 0.03$ obtained from [49–52]. By considering the radial excitation of $M$ (where $M = M_{B^*}$ or $M_{B_A^*}$), suggested by the dispersion relation for the $BK_1$ channel as indicated in (37), the value is $g_{B^* B K_1} = 15.42\,\text{GeV}^{-1}$. Also we have predicted the relationship between the $S$ and $D$ wave couplings $g_{B_A^* B K_1} = 3.17 f_{B_A^* B K_1}\,\text{GeV}^2$ given in (38). We have summarized our form factors in (44) and their values at $s = 0$ in (45). By using these form factors we have calculated the branching ratio for $B \to K_1 \mu^+ \mu^-$ both by considering the non-resonant and resonant value of the Wilson coefficient $C_9^{\text{eff}}(\hat{s})$, which will be seen in future experiments. The decay distribution is shown graphically in Fig. 1, where the differential decay rate is plotted as a function of $\hat{s}$.

A detailed analysis of the forward–backward asymmetry is also presented here. We have plotted the forward–backward asymmetry as a function of $\hat{s}$ in Fig. 2. It is clear from the graph that in the SM the central value of the zero of the FB asymmetry is at $\hat{s} = 0.16$ ($s = 4.46$). This value of the zero of the forward–backward asymmetry will provide a precision test of SM in planned future experiments.

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